

LET'S CONNECT PROBABILITIES TO GRAPHS!

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Causality Reading Group

Determinism

Axioms

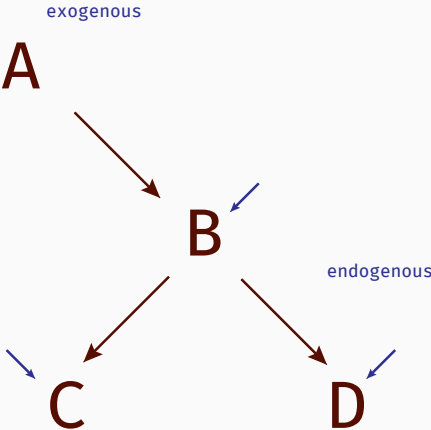
- Causal Markov Condition

- Causal Minimality Condition

- Faithfulness Condition

DETERMINISM

DETERMINISM



Three levels of determinism in a causal structure:

- **Deterministic** - endogenous variables fully defined by their parents (and thus, by the exogenous variables)
- **Indeterministic** - endogenous variables have random noise
- **Pseudoindeterministic** - There exists a graph over a superset of the variables that is deterministic but we don't know it.

“Structural equation models in the social sciences are usually assumed to be **pseudoindeterministic** causal structures.” (p. 28)

AXIOMS

CAUSAL MARKOV CONDITION

Let \mathbf{V} be a set of variables.

Let \mathbf{G} be a causal graph over \mathbf{V} .

Let \mathbf{P} be a probability distribution over \mathbf{V} .

\mathbf{G} and \mathbf{P} satisfy the **Causal Markov Condition** iff for every $W \in \mathbf{V}$,

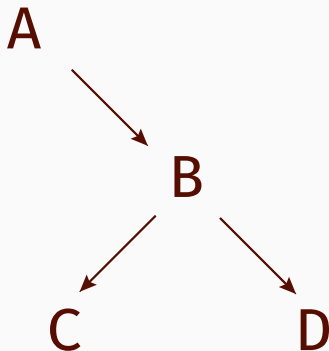
$$W \perp\!\!\!\perp \mathbf{V} \setminus \text{Descendants}(W) \mid \text{Parents}(W)$$

CAUSAL MARKOV CONDITION

$$C \perp\!\!\!\perp \{A, D\} | B$$

$$D \perp\!\!\!\perp \{A, C\} | B$$

$$B \perp\!\!\!\perp \emptyset | A$$



If the Causal Markov Condition holds, then we can factorise \mathbf{P} :

$$P(\mathbf{V}) = \prod_{W \in \mathbf{V}} P(W | \text{Parents}(W))$$

so in our example:

$$P(\mathbf{V}) = P(A)P(B|A)P(C|B)P(D|B)$$

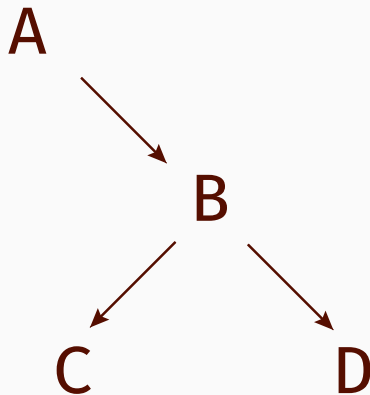
neat!

The grand axiom of:

“Don’t add arrows which are not supposed to be there”

Formally: for a given $P(V)$, if any **subgraph** of G over all of V exists for which the Causal Markov Condition holds, then G is not causally minimal.

CAUSAL MINIMALITY CONDITION



The big one:

" \mathbf{P} and \mathbf{G} are faithful to one another iff all independences in $\mathbf{P}(\mathbf{V})$ are implied by the markov condition applied to \mathbf{G} ."

" $\mathbf{P}(\mathbf{V})$ is faithful if a DAG \mathbf{G} exists to which it is faithful."

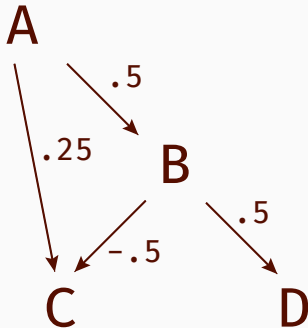
"Important to discovering causal structure".

Can we break faithfulness?

A WILD QUESTION APPEARS!

FAITHFULNESS CONDITION

Will a multivariate normal probability distribution generated from this linear pseudodeterministic causal structure be faithful to it?



I tested using R:

```
1 mod <- "  
2 B ~ 0.5*A  
3 C ~ -0.5*B + 0.25*A  
4 D ~ 0.5*B  
5 "  
6 d <- lavaan::simulateData(mod, sample.nobs = 1e7L)  
7 cor(d$A, d$C)  
8  
9 # [1] 0.000120153
```

Figure: Sorry for all the pixels, LaTeX was not cooperating...

