New Approaches To Analysing Psychological Time Series

SAA 2023 Symposium

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Extracting Dynamic Features from Irregularly Spaced Time Series

R package *expct*

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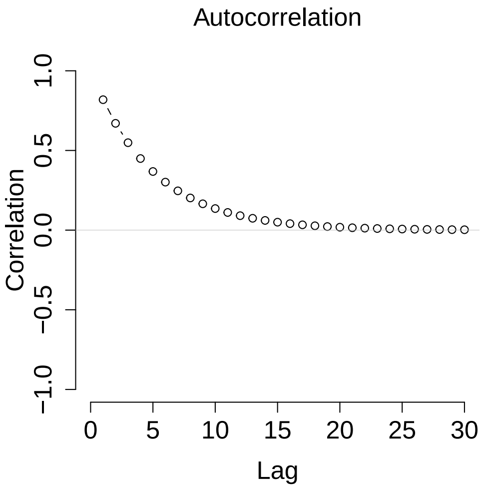
Descriptive / Exploratory Tools

- Autocorrelation function (ACF)
- Cross-Correlation function (CCF)

Y

y_1
y_2
y_3
y_4
y_5
y_6
y_7
y_8
$\vdots$
y_T

Y	Y at lag 1	Y at lag 2
y_1		
y_2	y_1	
y_3	y_2	y_1
y_4	y_3	y_2
y_5	y_4	y_3
y_6	y_5	y_4
y_7	y_6	y_5
y_8	y_7	y_6
$\vdots$	$\vdots$	$\vdots$
y_T	y_{T-1}	y_{T-2}
	y_T	y_{T-1}
		y_T



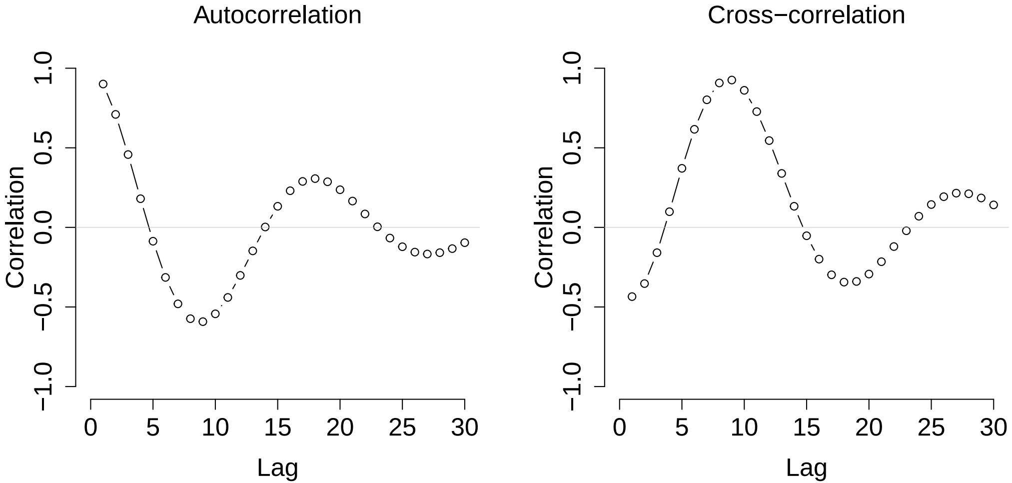
Autocorrelation
Correlation
1.0
0.5
0.0
−0.5
−1.0
0
5
10
15
20
25
30
Lag

Descriptive / Exploratory Tools

- Autocorrelation function (ACF)
- Cross-Correlation function (CCF)

Models such as VAR and ARIMA

- AR(1), AR(2), VAR(p)


Autocorrelation
Cross−correlation
Correlation
1.0
0.5
0.0
−0.5
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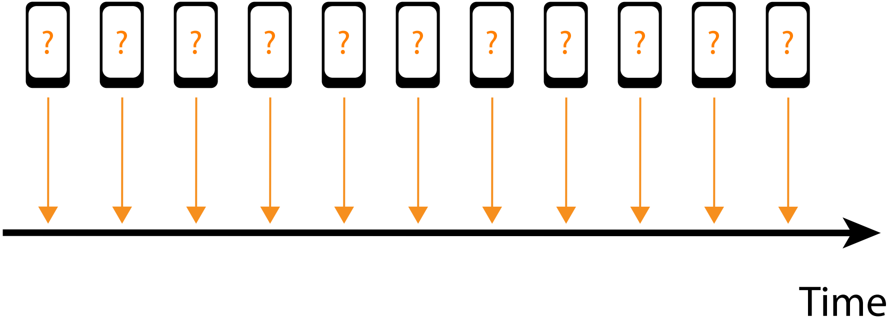
0
5
10
15
20
25
30
Lag

Assumption: Equally Spaced Measurements



Equally Spaced Data

Reality: Irregularly Spaced Measurements

Reality: Irregularly Spaced Measurements

Measurement Spacing in Empirical Data

Wichers & Groot (2016)

Unequally Spaced Data

Continuous-Time Modeling

Avoids time-interval problems by modelling moment-to-moment dynamics directly:

$$\frac{dY(t)}{dt} = A\,Y(t) + G\,\frac{dW(t)}{dt}$$

Re-written allows us to model variable relations at different time-intervals (Δt)

$$Y(t + \Delta t) = e^{A \Delta t}\,Y(t) + \zeta(\Delta t)$$

Can be estimated, e.g., using the ctsem package (Driver et al. 2017)

- Use $e^{A \Delta t}$ to inspect auto-and cross regression effects at different time-intervals
- Or transform to find model-implied auto and cross correlations

Boker (2002); Oud & Delsing (2010); Voelkle et al (2012); Ryan & Hamaker (2022)

Problem: Model Misspeci cation

Model-based correlations accurate only if the model is correctly speci ed

In reality the model is never correctly speci ed .

- We may have the order of the model (rst vs second) wrong
- Functional form of the relationships may not be linear at the DE level
- Even if we have a linear rst order model, if we have unobserved confounders or even mediators, we may run into problems

ctsem estimation: Simple Model

ctsem estimation: Misspeci ed Model

Traditional ACF estimation:

- Data-driven and exploratory method for exploring dynamic features
- Does not perform well with unequally spaced time series

Continuous-Time (CT) model estimation:

- Can be estimated from unequally spaced time series
- Inferences rely on correct model speci cation
- Without a data-driven way of computing auto- and cross- correlations: no easy way to check model misspeci cation

expct: Exploratory Continuous Time Modeling

Method to estimate ACF and CCFs from unequally spaced data

R package: github: ryanoisin/expct

expct: Exploratory Continuous Time Modeling

Method to estimate ACF and CCFs from unequally spaced data
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Two-step procedure

1. Create a \stacked" data frame

Y	Time stamp
y_1	0
y_2	s_1
y_3	s_2
y_4	s_3
y_5	s_4
y_6	s_5
y_7	s_6
y_8	s_7
$\vdots$	$\vdots$
y_T	s_T

expct: Exploratory Continuous Time Modeling

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Two-step procedure

1. Create a \stacked" data frame
2. Use Generalized Additive Model (GAM) to estimate how lagged correlations depend on the time-interval

$$Y_{t+\ t} = f(\ t)Y_t +$$

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Two-step procedure

1. Create a \stacked" data frame
2. Use Generalized Additive Model (GAM) to estimate how lagged correlations depend on the time-interval

$$Y_{t+\ t} = f(\ t) Y_t +$$

By rescaling $f(\ t)$ we estimate

$$\mathrm{cor}(Y_t; Y_{t+\ t}) / \ f(\ t)$$

expct estimation: unequally spaced

Simulation Study

Time-series length: [50 - 2000]
Sampling Scheme: f Equal, Unequal Bimodal, Unequal Uniform g
Data-generating models: f Simple, Oscillating, Complex (missing variables) g

Methods for computing CIs:

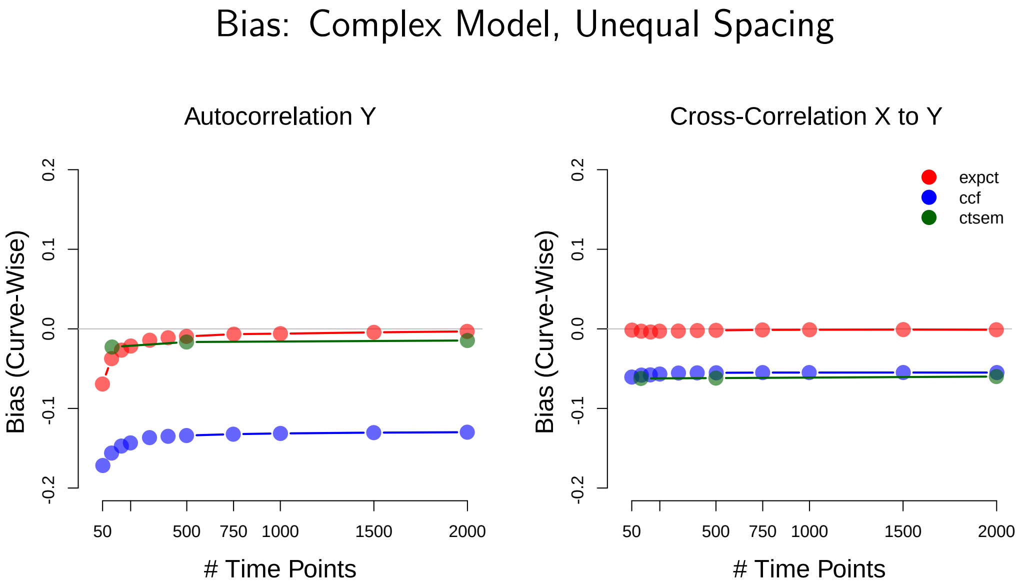
- Point-wise, Simultaneous, Analytic

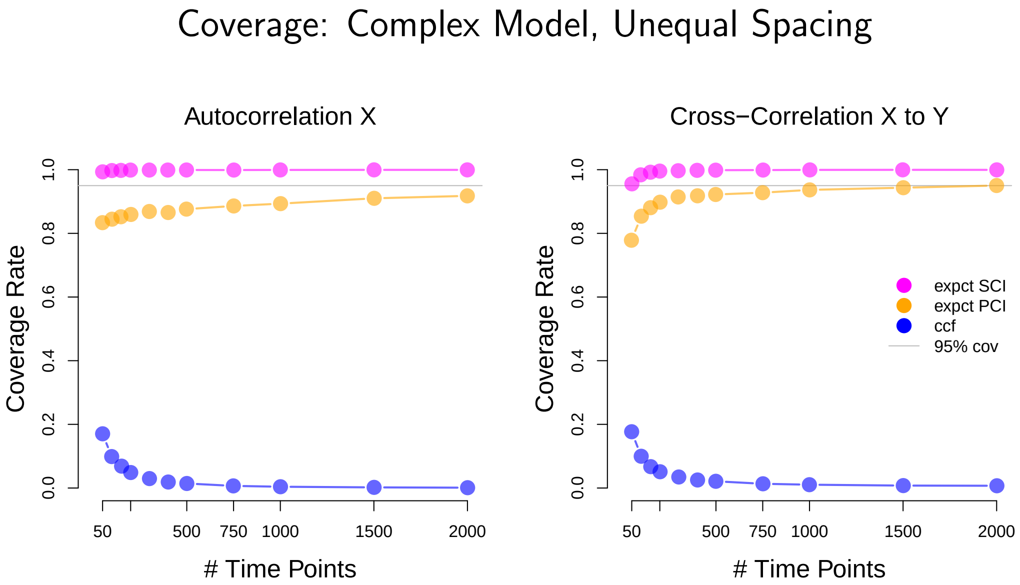
We use \function-wide" averages to summarize performance:

- Function-wide bias: Average distance between true and estimated correlation function evaluated at a range of time-intervals / lags

Bias: Simple Model, Equal Spacing

Bias: Complex Model, Equal Spacing

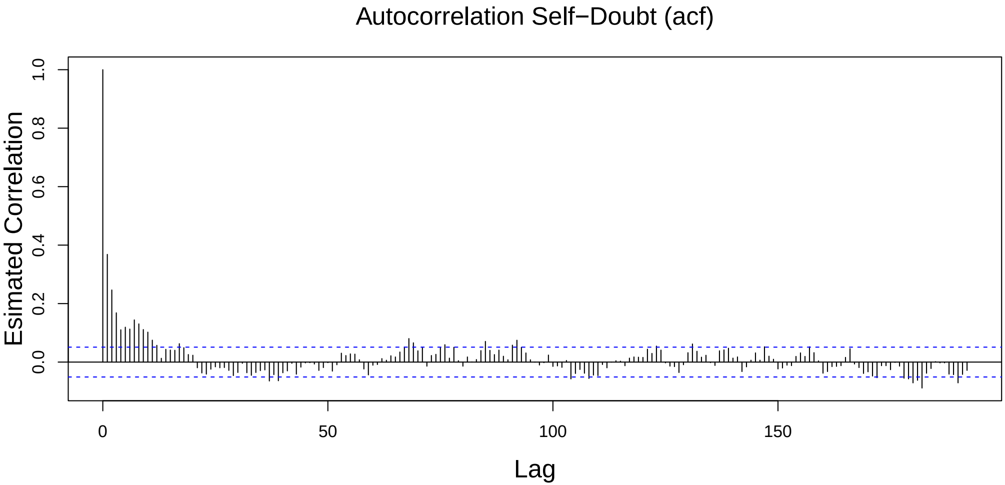

Bias: Complex Model, Unequal Spacing
Autocorrelation Y
Cross-Correlation X to Y
Bias (Curve-Wise)
0.2
0.1
0.0
-0.1
-0.2
50
500
750
1000
1500
2000
Time Points
expct
ccf
ctsem


Coverage: Complex Model, Unequal Spacing
Autocorrelation X
Cross−Correlation X to Y
Coverage Rate
0.0
0.2
0.4
0.6
0.8
1.0
50
500
750
1000
1500
2000
Time Points
expct SCI
expct PCI
ccf
95% cov

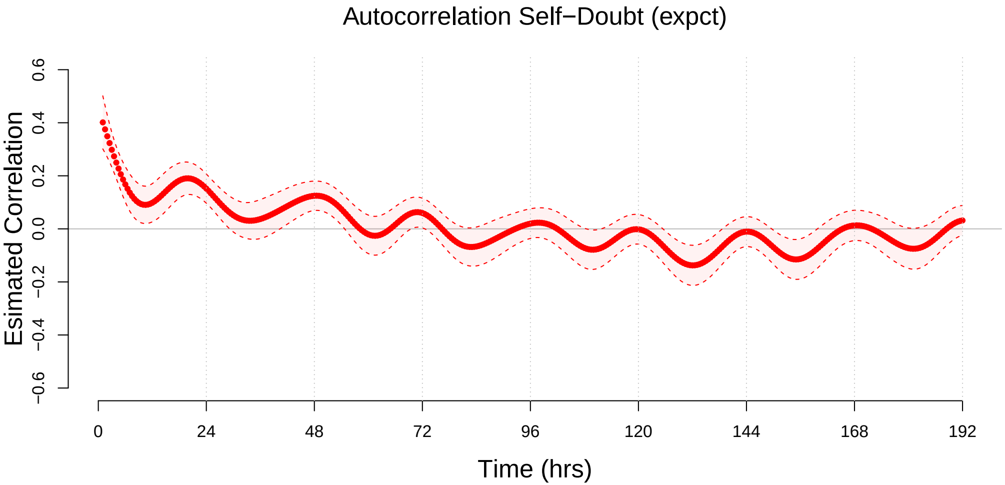
Empirical Data

Wichers & Groot (2016)

Empirical Data



Empirical Data



Future work

In principle this method can be used in other situations than those studied here

- Systems of variables measured at different timescales (e.g., daily diary vs hourly ratings vs minute-to-minute physiological measurements)
- "Panel" data: multi-subject low repeated measures

Extensions in progress:

- Multi-level time-series data (random effects)
- Partial relationships (PACF, PCCF)

Extracting Dynamic Features from Irregularly Spaced Time Series

expct: Exploratory continuous-time modeling

- Available as an R package github: ryanoisin/expct
- Overcomes equal-interval limitation of traditional ACF/CCF estimation
- Avoids reliance on correct lagged model specification in confirmatory continuous-time models
  - *ctsem*, *dynr*

Ryan O., Wu, K., & Jacobson, N.K. (in preperation). Exploratory Continuous-Time Modeling (expct): Extracting Dynamic Features from Irregularly Spaced Time Series

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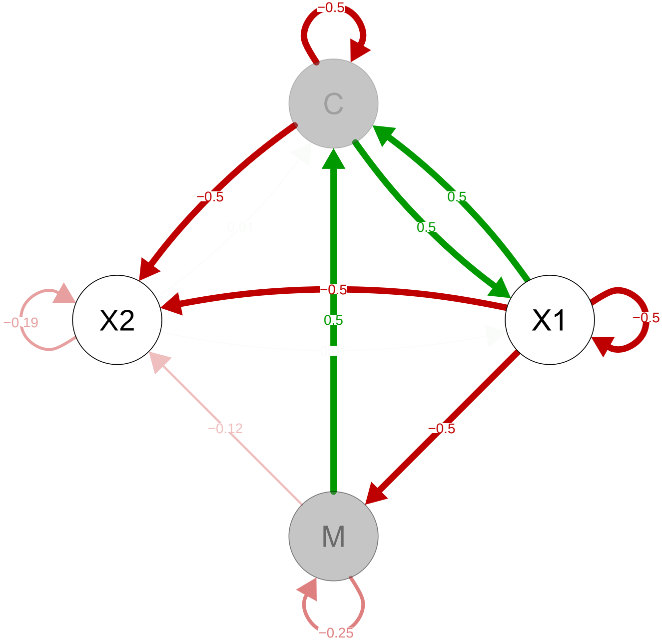

C
X2
X1
M
−0.5
−0.5
0.5
0.5
−0.5
0.5
−0.19
−0.5
−0.12
−0.5
−0.25